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Perfectly matched layer for the solution of acoustic wave equation using finite elements

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Abstract

In order to avoid boundary reflection when doing forward modelling, an artificial added absorbing boundary is necessary. The perfectly matched layer (PML) has become a popular and efficient way to solve this problem in seismic wave modelling. Here we focus on the application of this technique to finite-element frequency domain numerical modelling of acoustic wave equations in escript.

Introduction

The aim of this project is to complete a stochastic full waveform inversion (FWI) in the frequency domain. The first step is to solve the wave equation in the frequency domain for forward modeling. Some points of interest are:

- Numerical modelling in the frequency domain is flexible and of high computational efficiency.
- The finite element method is well suited for complex conditions and irregular grids.
- In order to eliminate artificial boundary reflections, PML (Berenger 1994) can produce zero reflection coefficients for all angles of incidence and frequencies without spurious reflections.
- The whole process is implemented in the escript modeling environment, which is easier to operate and calculate (Gross and Kemp 2013).

Method

Scalar acoustic wave equations in the frequency domain can be transformed via fourier transformation from second-order scalar acoustic wave equation in the time domain:

$$Au = -\nabla^2 u - k^2 u = g \qquad (1)$$

∇^2represents Laplace operator, u is displacement vector, $k = \omega/c$ is wavenumber,ω is angle frequency, c is velocity, g is source term.

For PML, we add an attenuation factor to the above equation as follows:

$$\tilde{A}u = \tilde{g} \qquad (2)$$

$\tilde{A} = -\left(\frac{1}{\gamma_x}\frac{\partial}{\partial_x}\left(\frac{1}{\gamma_x}\frac{\partial}{\partial_x}\right) + \frac{1}{\gamma_z}\frac{\partial}{\partial_z}\left(\frac{1}{\gamma_z}\frac{\partial}{\partial_z}\right)\right) - \frac{\omega^2}{c^2}$ represents a large sparse matrix, $\tilde{g} = \delta(t)$, $\gamma_\alpha = 1 - j\eta_\alpha/\omega$.

In terms of the value of $\eta_{\alpha,}\eta_\alpha = [D_{\alpha-PML}/L_{PML}]^2, \alpha \in (x,z),$ the numerical area can be divided into several parts. $D_{\alpha-PML}$ means the distance of grids from absorbing boundary to the α direction layer within the zone of absorbing, L_{PML} represents the width of the boundary. The computational region of η_α is shown in Figure 1.

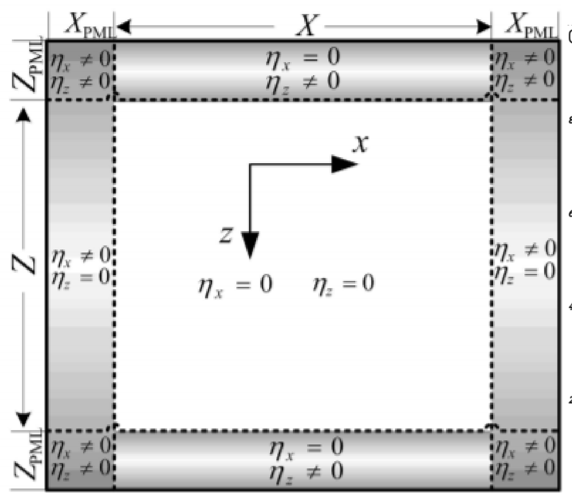

Fig. 1: PML attenuation factor.

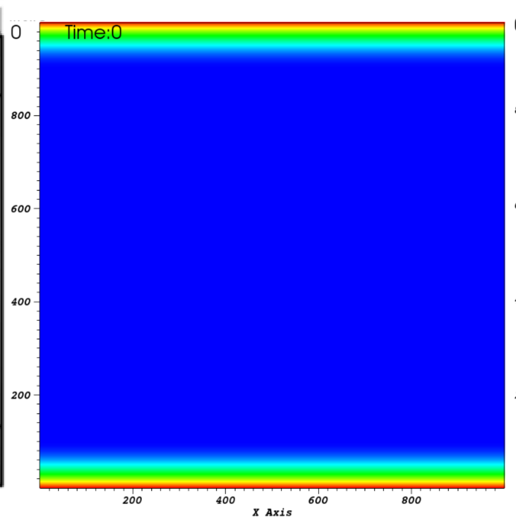

Fig. 2: Absorbing layer in x direction.

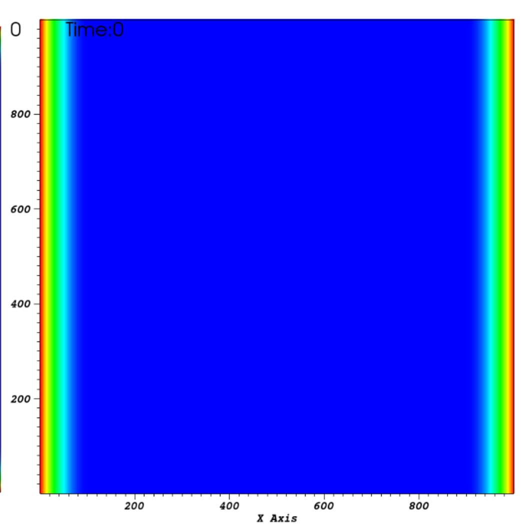

Fig.3: Absorbing layer in z direction

Numerical result

We use the escript modeling environment in python to implement wave propagation within the PML condition. The spatial step is 2m, the velocity of the media is 2000m/s, the source is the Ricker wave, the dominant frequency of wave is 20Hz, the time sampling interval is 0.005s. The numerical model area is 1000x1000m, the number of grids for PML is 15. We then put the source into the center of area, and deployed 15 geophones away from source horizontally. The offset was 100m, and the shot interval was 20m. Here is the signal from the first geophone.

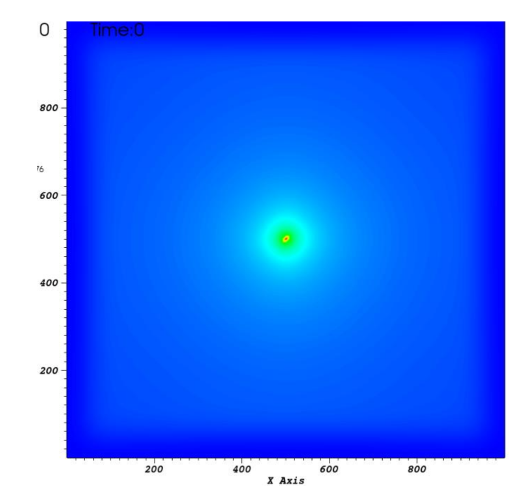

Fig.4: wavefield shot in frequency domain

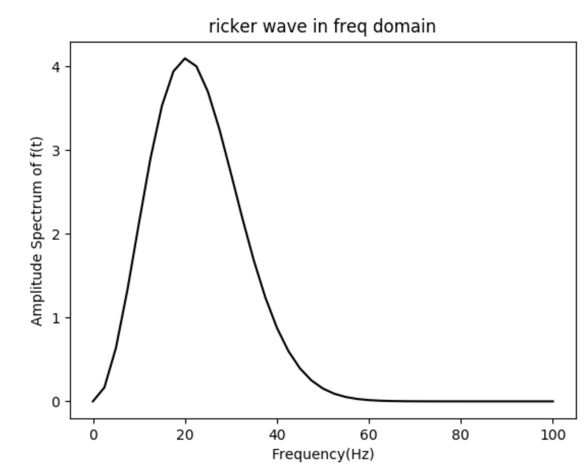

Fig.5: Ricker wave in frequency domain

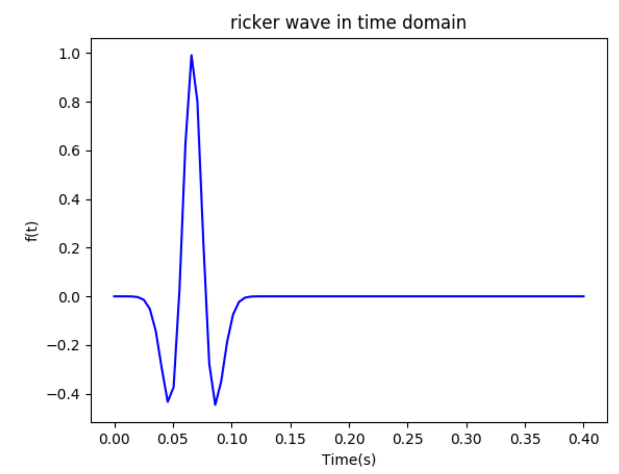

Fig.6: Ricker wave in time domain

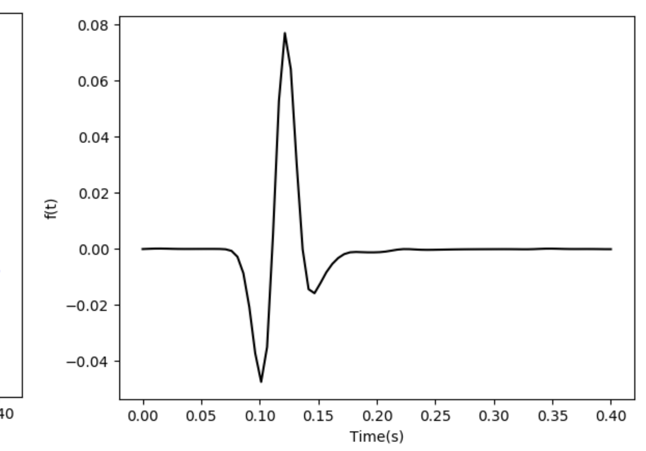

Fig.7: Signal transformed back from first geophone

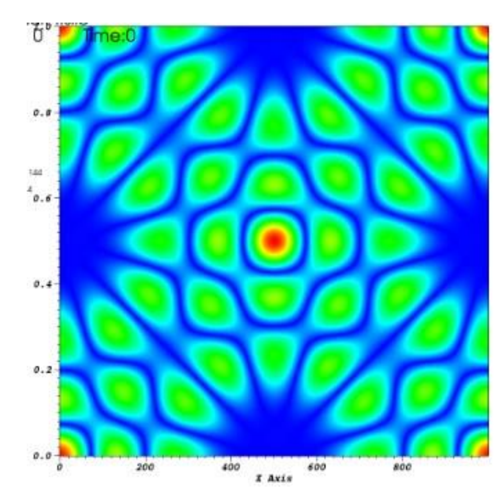

Fig.8: Wavefield without PML

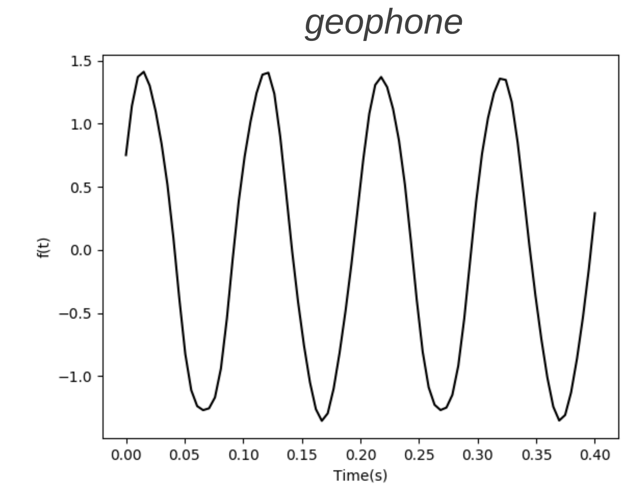

Fig.9: Signal from first geophone without PML

References

Berenger JP (1994), A perfectly matched layer for the absorption of electromagnetic waves, *Journal of Computational Physics*, 114(2), 185-200.
Gross L& Kemp C (2013), Large Scale Joint Inversion of Geophysical Data using the Finite Element Method in escript, ASEG Extended Abstracts, 2013:1, 1-4.