Damage and fracture analysis of rock under uniaxial compression

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Research team

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Objectives

- Numerical modelling of rock with pre-existing flaw.
- Random sampling of elastic modulus with Weibull.
- Smoothing damage over a localization length scale.
- Eliminating well known mesh dependency problem.
- Finite element method, parallel computing, 55 Million cells, *esys-escript* (Gross *et al.* 2015).

Domain and geometry of the model

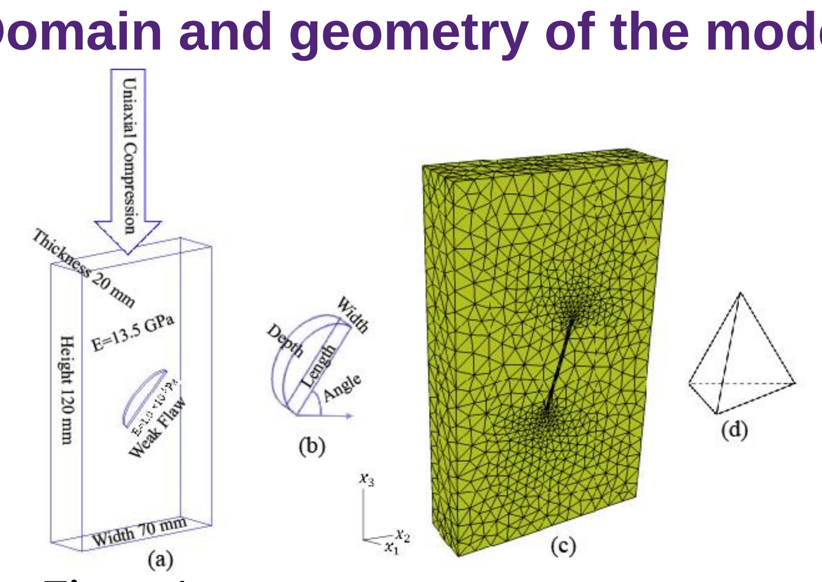


Figure-1: Numerical specimen for the simulation.

Governing equations

$$-\sigma_{ij,j} = 0 = -(C_{ijkl}u_{k,l})_{,j}$$

$C_{ijkl} = \frac{E\nu}{(1+\nu)(1-2\nu)}\delta_{ij}\delta_{kl} + \frac{E}{2(1+\nu)}(\delta_{ik}\delta_{jl}+\delta_{jk}\delta_{il})$ is the stiffness tensor.

Local damage model

The elastic modulus of the damaged model is defined as (Mondal *et al.* 2014),

$$E = E_0(1-D)$$

E is the Young's modulus random sampling from Weibull distribution and D is the damage variable.

$$D = \begin{cases} 1-\left(\frac{\kappa_0}{\kappa}\right)^{\beta}\left(\frac{\kappa_c-\kappa}{\kappa_c-\kappa_0}\right)^{\alpha} & if\ \kappa < \kappa_c \\ 1 & if\ \kappa \ge \kappa_c \end{cases}$$

$$\kappa = \frac{\gamma-1}{2\gamma(1-2\upsilon)}I_1 + \frac{1}{2\gamma}\sqrt{\frac{(\gamma-1)^2}{(1-2\upsilon)^2}I_1^2 - \frac{12\gamma}{(1+\upsilon)^2}J_2}$$

Where, $I_1 = \varepsilon_{kk}$, $J_2 = \frac{1}{6}I_1^2 - \frac{1}{2}\varepsilon_{ij}\varepsilon_{ij}$ and $\gamma = \frac{\sigma_c}{\sigma_t}$, which is the ratio the compressive and tensile strength. κ is the equivalent strain.

Nonlocal damage model

Helmholtz smoothing equation: $\bar{\varepsilon} - c\nabla^2\bar{\varepsilon} = \kappa$

Where, $c = \frac{1}{2}l^2$ and l is the localization length scale which represents the smoothing region of damage.

Model validation with concrete specimen

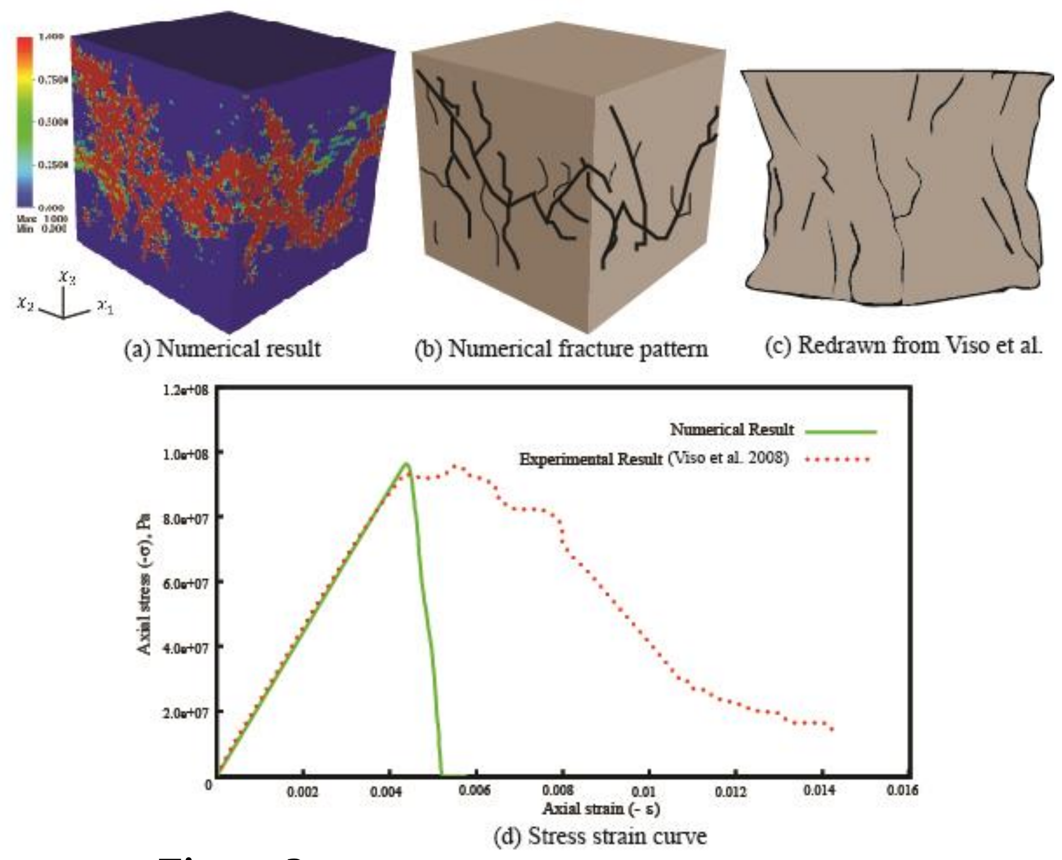
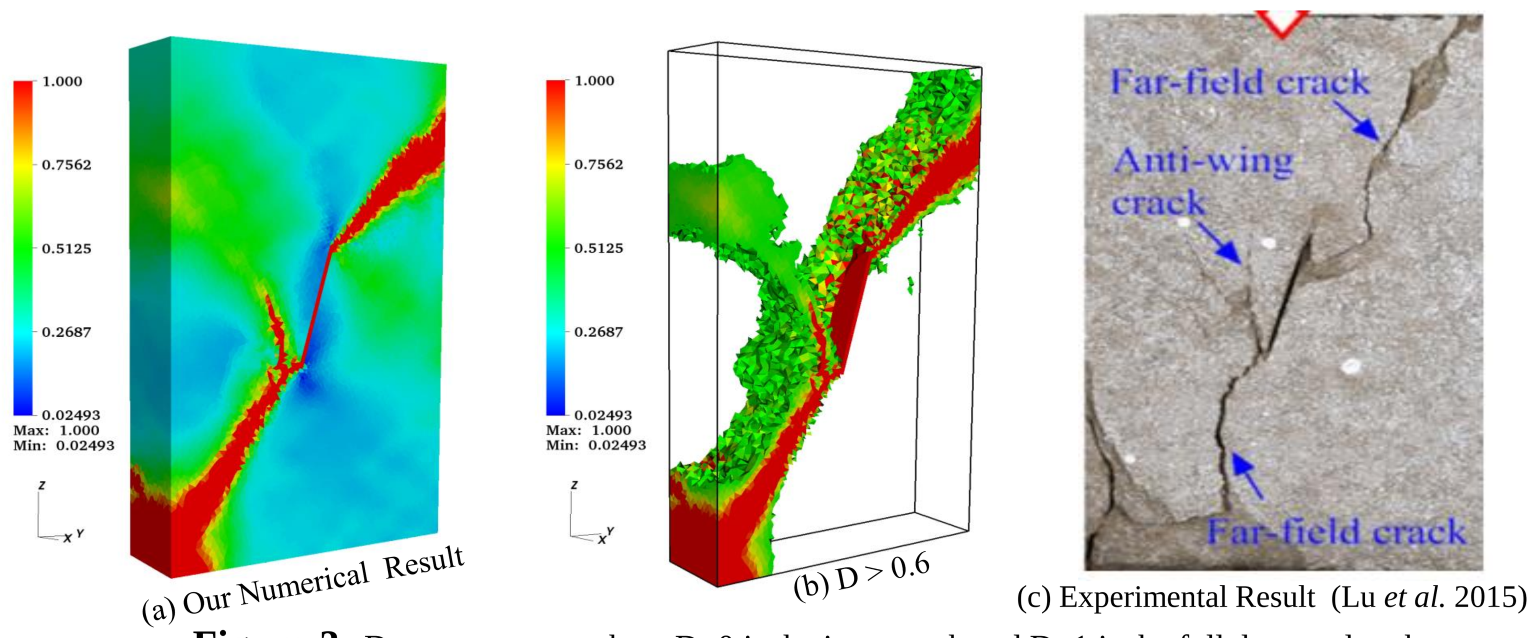


Figure-2: Numerical validation for the cubic specimen.

Our numerical results have good agreement with the experimental results of Viso *et al.* (2008) for both damage pattern and the peak stress of a 100 mm^3 cubic concrete specimen.

Application in a pre-existing sandstone specimen



(c) Experimental Result (Lu *et al.* 2015)

Figure-3: Damage pattern where D=0 is the intact rock and D=1 is the full damaged rock.

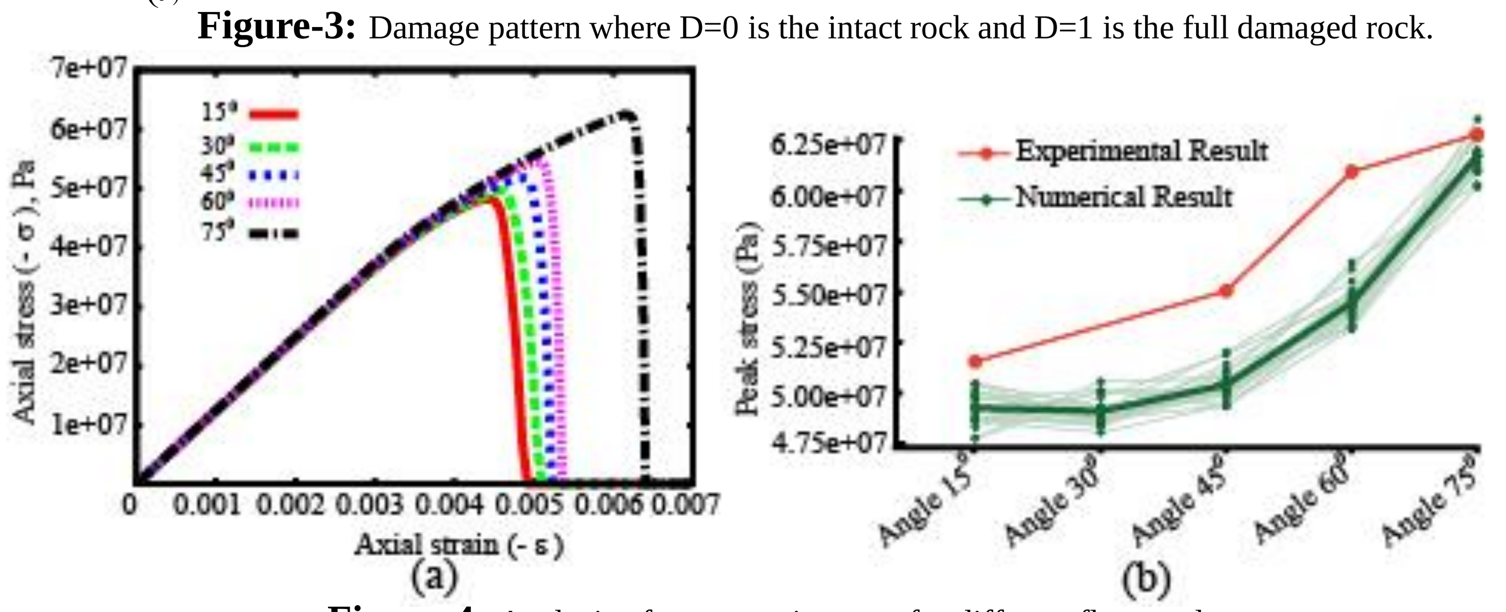


Figure-4: Analysis of stress strain curve for different flaw angles.

Our numerical results in Figure-3 have good agreement with the experimental result of Lu *et al.* (2015) for a sandstone specimen with a pre-existing surface flaw under uniaxial compression. The peak stress is decreasing with the decreasing value of the flaw angle in Figure-4. Different realisation of elastic modulus represent the statistical variation in peak stress up to 15% which is a quite good result.

Local and nonlocal damage model

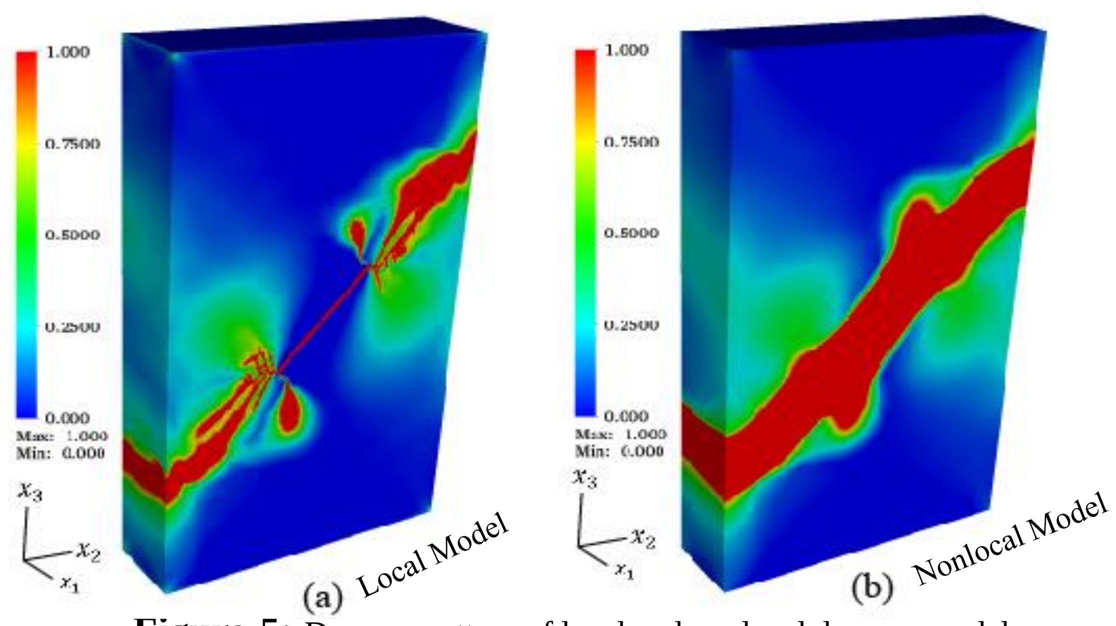


Figure-5: Damage pattern of local and nonlocal damage model.

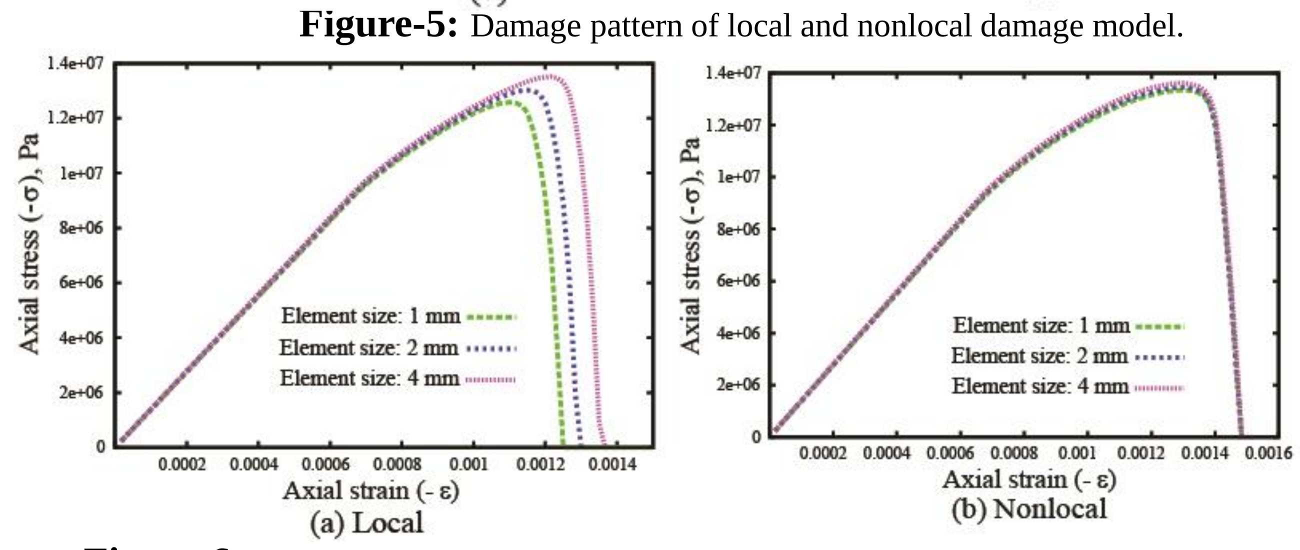


Figure-6: Mesh dependency analysis of local and nonlocal damage model, where l=8 mm.

The damage pattern of the nonlocal model in Figure-5 is wider than the local model because of the large localisation length scale. If we used a smaller localisation length in the nonlocal model we could produce a damage pattern on a similar scale to the experimental result. However, smaller localisation lengths require a much finer mesh resolution for the *FEM* mesh and this is very computationally costly. From Figure-6 it is clear that nonlocal model can remove mesh dependence.

Conclusion

The objective of the research was to develop a mathematically consistent continuum damage formulation which can describe the fracture propagation and damage evolution from a pre-existing fracture of a 3-D rock specimen under uniaxial compression. From this study, it is concluded that the non-local implicit gradient damage formulation can remove the pathological mesh dependence and strain localisation in local damage models. This nonlocal damage model introduces an additional length scale: the localization length, which must be adequately resolved by the finite element mesh and places a constraint on the mesh resolution. Both the local and nonlocal damage model can be used to model rock fracture and burst in mining, and well-bore stability in oil and gas operations. Other applications of these damage models of brittle failure are in composites and concrete.

References

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Acknowledgements

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Research with real world impact