Non-Linear Geostatistics

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1. Motivation

Fluid flow in porous media is characterised by the extreme high and low permeability regions and in particular by their continuity. Existing geostatistical algorithms in commercially available software are based on the Gaussian assumptions. In general, a Gaussian spatial dependence implies a low spatial correlation of extremes, resulting in an underestimation of the connectivity of high and low permeability regions. Multiple point statistics (MPS) are frequently applied to model non-Gaussian spatial dependence structures. Their main drawback however is the necessity for a suitable training image.

The approach suggested here is based on random mixing of spatial random fields and uses spatial copulas to model the spatial dependence. The main advantages are:

1. A high flexibility to handle several kinds of conditioning constraints.
2. The description of the spatial dependence using spatial copulas, which enables non-Gaussian spatial structures. As the copula model is fitted to the observed data directly, there is no need for a training image. However, if a suitable training image is available the suggested approach can be coupled to MPS.
3. The possibility to handle non-linear constraints, e.g. hydraulic observations.

Problems with conventional geostatistics

Estimation variance is an index of spatial configuration

1. Does not depend on the local values
2. "Best" for Gaussian distribution
3. Symmetrical (high and low values not distinguished)

Variogram estimation difficult

1. Squared differences – skewed distribution
2. Dominated by high values
3. Strongly influenced by the marginal distribution

Kriging Variance doesn't capture true variance

$$\sigma_K^2(u) = \sum_{j=1}^{n} \lambda_j \gamma\left(u_i - u_j\right) + \mu$$

2. Theory

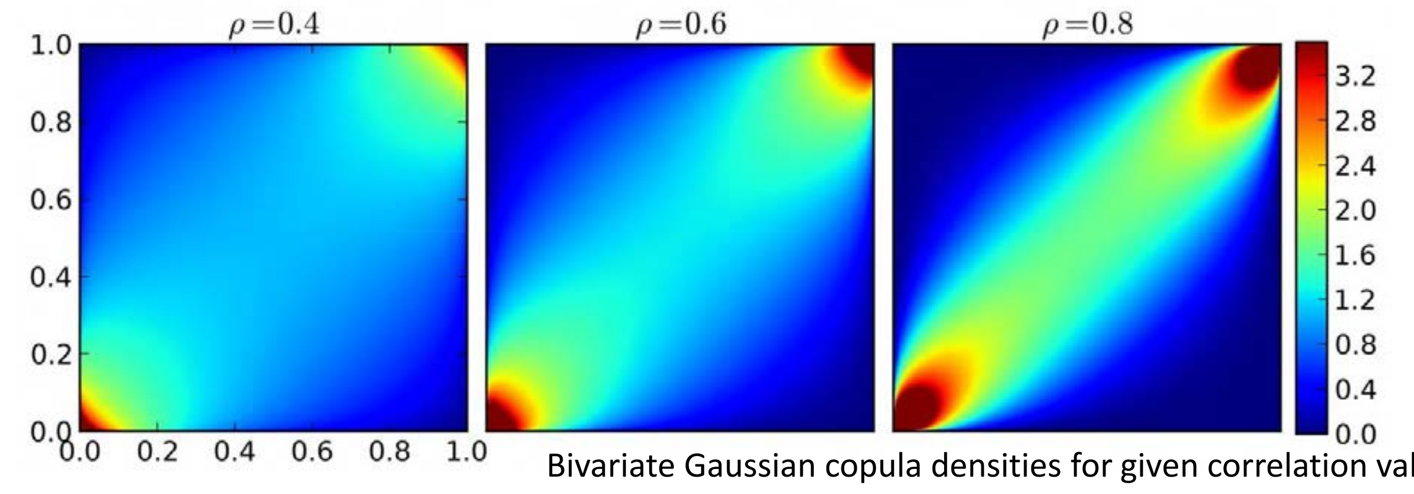
Using random mixing a conditional realisation *Z(x)* is constructed as a linear combination of unconditional random fields *Y(x):*

$$Z(x) = \sum_{i=1}^{n} \alpha_i Y_i(x)$$

where the fields Y(x) have zero expectation and unit variance, and the weights are chosen such that:

$$\sum_{i=1}^{n} \alpha_i Y_i(x_t) = z_t \quad t = 1, \ldots, T \qquad \sum_{i=1}^{n} \alpha_i^2 = 1$$

The spatial dependence of *Z(x)* is described with the help of a spatial Gaussian copula. In general, copulas are multivariate distributions that describe the dependence between random variables independently of their marginal distributions, thus arbitrary marginal distributions can be considered *[Nelson, 1999]*. Nevertheless, the flexibility of the Gaussian copula is restricted by its symmetry properties.

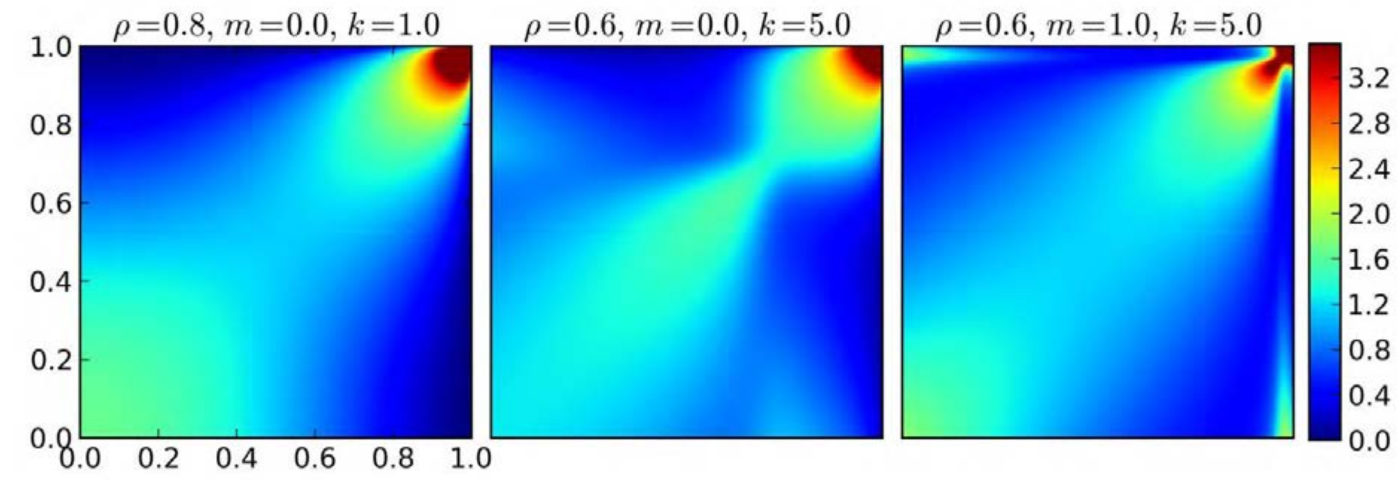


Bivariate Gaussian copula densities for given correlation values ρ.

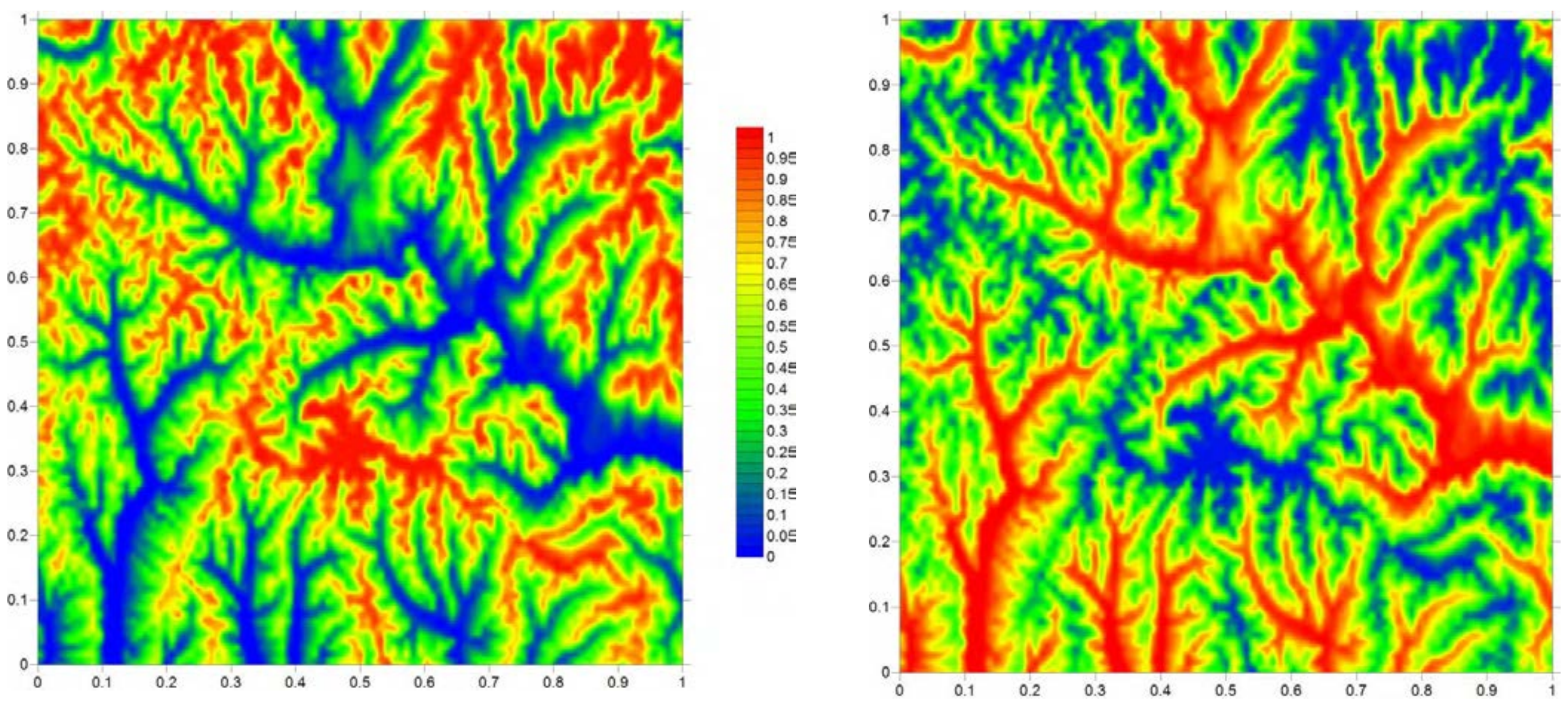
Copulas that are able to describe asymmetric dependence structures can be obtained by non-monotonic transformations of the multivariate Gaussian distribution. An example is the v-transformation given in Bárdossy and Li (2008).

As this transformation changes the ranks of the variables, the resulting v-copula is no longer symmetrical.

As the v-copula is based on a non-monotonically transformed multivariate normal distribution it can be applied for random mixing. Thus random fields with asymmetrical dependence structure, i.e. connected features of high and low values can be achieved. Fig. 3 shows fields obtained from different copula models. The difference between the Gaussian and non-Gaussian case is clearly visible.

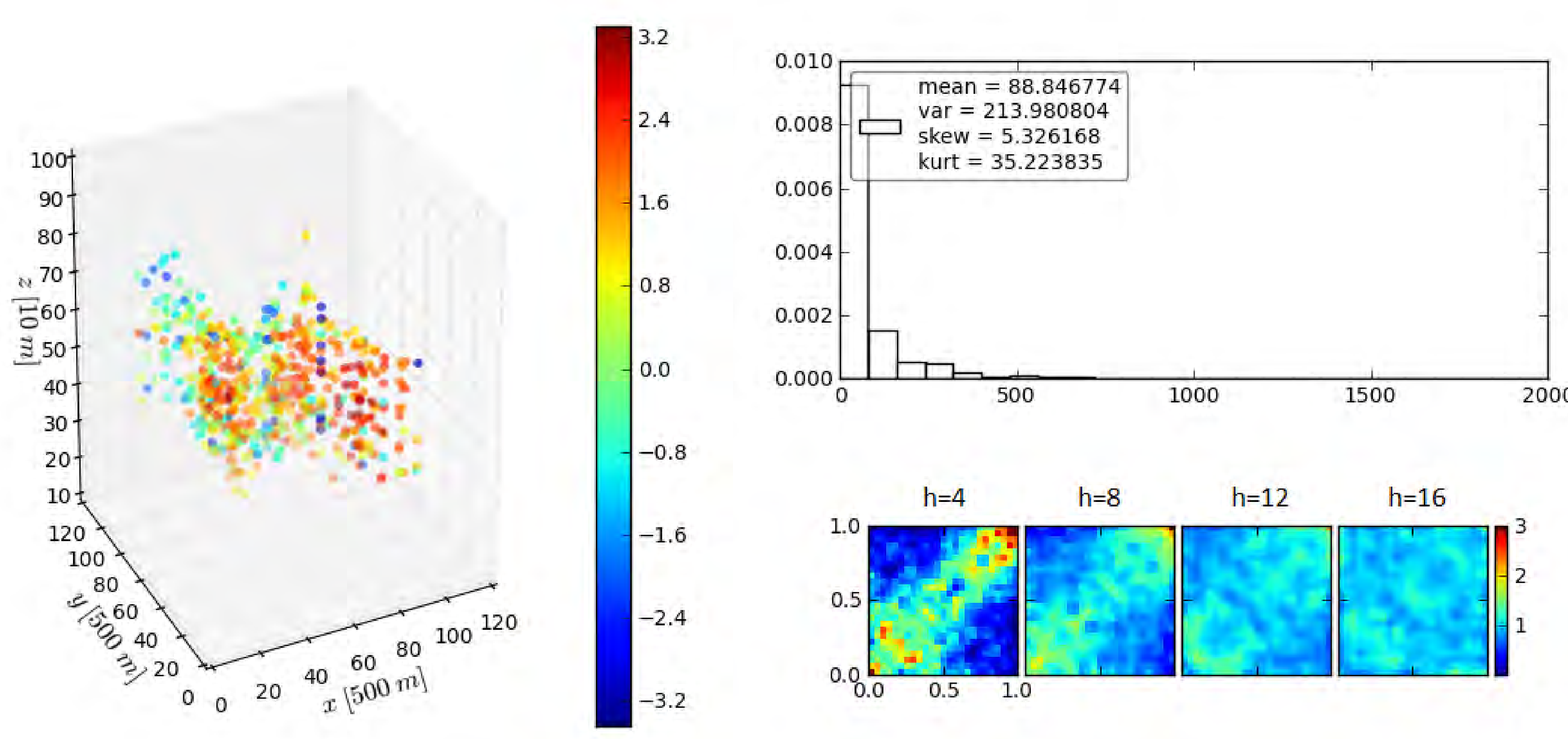


3. Which one is upside down?



Surface elevation – red is high, blue is low. Which one is upside down?

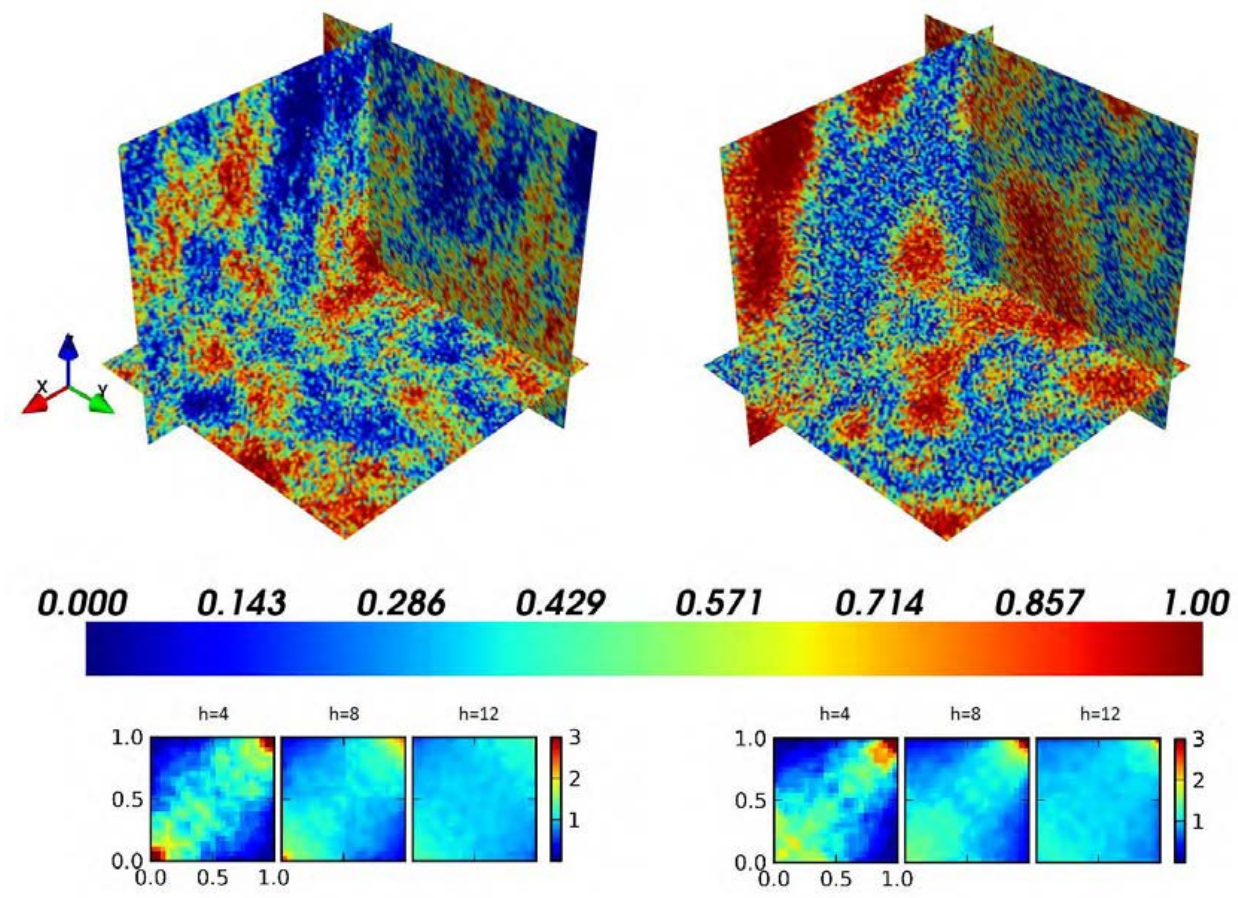
4. Application to permeability data



Empirical spatial statistics of a permeability data set. The left plot shows the spatial configuration of permeabilities [log10 mD], the lower right shows the corresponding empirical copulas for given separation distances. The upper right shows the histogram of the measured permeabilities [mD]. It can be seen that the empirical copulas differ from Gaussian indicating an asymmetrical spatial dependence structure

In order to generate conditional realizations of the given data set via random mixing, a Gaussian copula as well as a v-copula is fitted to the data set using maximum likelihood. Kernel density estimation is applied to fit a non-parametric marginal distribution to the observed permeabilities. Thus the first case exhibits a multiGaussian spatial dependence; the second case exhibits a non-Gaussian spatial dependence. The marginals are the same in both cases.

One conditional realization for each copula model with corresponding empirical copulas is shown below. It can be seen that the empirical copulas corresponding to the field based on the v-transformed copula are similar to those of the permeability data set. The empirical copulas corresponding to the field based on the Gaussian copula are symmetrical. Thus the v-copula is able to represent the spatial dependence structure of the given data, while the Gaussian copula fails.



Cross-sections of conditional (conditioned on 627 observed values) 3-dimensional permeability fields (in copula space) with corresponding empirical copulas. The left field corresponds to the Gaussian copula, the right field to the v-transformed copula.

5. Conclusion

This work shows how asymmetrical, non-Gaussian spatial dependence structures can be modeled using a copula-based simulation approach called random mixing. The main advantage compared to MPS is that a copula is fitted directly to the observed data hence there is no need for a suitable training image. However, recent studies showed that the presented approach can be coupled to MPS. Such a coupling is especially useful if for example a geological map corresponding to the data of interest is available.

The suggested approach is able to reasonably quantify the associated estimation uncertainty and different kinds of linear conditioning constraints, e.g. inequalities can be considered. Further, random mixing was tested in an inverse modeling framework, i.e. the fields were additionally conditioned on hydraulic observations. The results obtained are quite promising.

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